EPR test with photons and kaons: Analogies

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We present a unified formalism describing the Einstein–Podolsky–Rosen test using spin $\frac{1}{2}$ particles, photons, and kaons. This facilitates the comparison with existing experiments using photons and kaons. It underlines the similarities between birefringence and polarization-dependent losses that affect experiments using optical fibers and mixing and decay that are intrinsic to the kaons. We also discuss the limitation these two characteristics impose on the testing of Bell's inequality.

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I. INTRODUCTION

Quantum mechanics is one of the most successful physics theories of the 20th century. Over the past 70 years since its inception, quantum theory has been tested in many areas of physics with high precision. However, its radical departure from everyday intuition has troubled physicists for the past 70 years. One of the most puzzling predictions is entanglement: the fact that a multiparticle wave function automatically implies nonlocal correlation when the constituent particles are spatially separated. This radical departure from our intuitive understanding of nature has prompted hot debates over the years.

II. THE EPR–BOHM EXPERIMENT

In 1935, Einstein, Podolsky, and Rosen (EPR)[1,2] proposed a thought experiment involving two spatially separated particles with which they aimed to illustrate the incompleteness of the then newly developed quantum theory (QM). Today their argument is no longer considered to imply incompleteness of QM, but rather to imply that one of their hidden assumptions is wrong.

The original discussion of this thought experiment was in terms of momentum and position, but a simpler variant was given by Bohm[3] using two distant spin-$\frac{1}{2}$ particles in the singlet state (Fig. 1). The spin part of the wave function is given by

$$|\psi\rangle=\frac{1}{\sqrt{2}}[|+\mathbf{m}\rangle_1\otimes|-\mathbf{m}\rangle_2-|-\mathbf{m}\rangle_1\otimes|+\mathbf{m}\rangle_2], \quad (1)$$

where $|\pm\mathbf{m}\rangle_j$ describes a state of the jth particle ($j=1,2$) with spin up or down along the direction $\mathbf{m}$, respectively. According to QM, if one measures the spin along, say, the $\hat{x}$ axis, the outcome is not predetermined by the wave function; but whatever result one obtains, the spin of particle 2 will always be antiparallel to the spin of particle 1. The EPR argument goes as follows.

• First, the predictions of QM concerning observation of the two spatially separated particles are correct.

• Next, Nature is local. This point was not spelled out precisely, because it seemed obvious to EPR. It is implied in the sentence, "there is no longer any interaction." The general idea is that one cannot influence anything "there" by acting "here," when "here" and "there" are disconnected, in particular when "here" and "there" are space-like separated.

• Third, "If without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity."

• Consequently, since one can measure either $S_{\hat{x}}$ and $S_{\hat{y}}$ on particle 1, there are "elements of reality" corresponding to the physical quantities $S_{\hat{x}}$ and $S_{\hat{y}}$ on particle 2. On the other hand, since QM does not describe such elements of reality, EPR concluded that it must be incomplete.

The first point of the EPR argument is a matter of experimental evidence. In 1935 and even when Bell formulated his inequality in 1964,[4] there was little evidence that QM correctly describes such two-particle distant systems. But today there is plenty of evidence in favor of QM. The third point of the EPR argument received a lot of attention in the vast literature on the subject. However, we view it as a complicated way of stating the obvious assumption implicit in all natural sciences according to which, if one can predict the result of an experiment with 100% certainty, then the system out there has the property corresponding to this predetermined result (i.e., if we know the result in advance, Nature knows it also). Note that EPR did not assume the existence of "elements of reality" (or properties of physical systems, or name it using your preferred terminology); rather they inferred their existence from the general concept of locality and by assuming that QM describes distant systems correctly.

At this point, the EPR argument leaves two alternatives open: Either QM is incomplete (as concluded by EPR) or Nature is nonlocal (the only remaining assumption, point 2 above). However, thanks to Bell's inequality, one can prove (though there is still a logical loophole, called the detection loophole[5]) that Nature is nonlocal (in a precise sense to be described below). Note that this does not prove the completeness of QM: It only establishes that the EPR argument is not valid because its second assumption, the one that Einstein and his co-authors found so self-evident that they did not spell it out explicitly: the locality assumption, is wrong.

The sense in which Nature is nonlocal is not easy to grasp. Let us first stress that it does not imply any direct conflict with relativity: There is no controllable action at a distance,

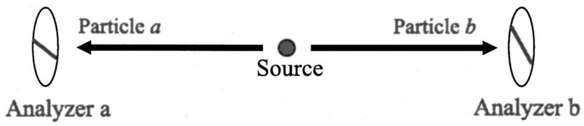

Fig. 1. Schematics of the entangle pair of spin-$\frac{1}{2}$ particles with the polarization analyzers.

hence no way to use it for faster than light communication. Indeed, according to QM, the result of any measurement on a quantum system is always independent of any parameter or setting chosen to measure the other particle. Jarrett[6] and Shimony[7] have named this important characteristic *parameter independence*. However, the result on one particle may depend on the result obtained on the other particle. The difference with classical correlation is that the result cannot be assumed to pre-exist. In Jarrett and Shimony terminology, this is called *outcome dependence*. Henceforth, we call this kind of nonlocality (outcome dependence but parameter independence) *quantum nonlocality*. It is plausible that if Einstein had realized that quantum nonlocality does not contradict relativity, he would have revised his argument, though this will remain questionable forever.

Many experimental tests have been carried out over the years, mostly in two-photon experiments with polarization entanglement.[8–10] Recently, the group of the first author (NG) has performed an experimental test with photons having time–energy correlation traveling through optical fibers over a distance of 10 km.[11] The second author (AG) has participated in another test of entanglement using massive K mesons and the strangeness correlation has been measured.[12] In this paper we will discuss the similarity between kaon/anti-kaon correlation in time evolution and the photon polarization correlation in fibers.

III. EPR TEST USING PHOTON POLARIZATION AND OPTICAL FIBERS

Most experiments have been done with photons. Hence we recall first the analogy between the polarization of photons and spin $\frac{1}{2}$.

A. Single photon polarization formalism

Since any pure spin-$\frac{1}{2}$ state is represented by a normalized vector in our familiar three-dimensional space, the set of pure spin-$\frac{1}{2}$ states is naturally represented by the unit sphere, called the Bloch sphere.

Any pure polarization state $|\mathbf{m}\rangle$ can also be represented geometrically by a point on an abstract sphere called the Poincaré sphere. The corresponding normalized vector with its associated (normalized) spinor are

$$\mathbf{m}=(\sqrt{1-\eta^2}\cos(\varphi),\quad \sqrt{1-\eta^2}\sin(\varphi),\eta) \tag{2}$$

and

$$|\mathbf{m}\rangle=\begin{pmatrix}\sqrt{\dfrac{1+\eta}{2}}e^{-i\varphi/2}\\ \sqrt{\dfrac{1-\eta}{2}}e^{+i\varphi/2}\end{pmatrix}, \tag{3}$$

respectively. The one-dimensional projector reads

$$P_{\mathbf{m}}=|\mathbf{m}\rangle\langle\mathbf{m}|=\frac{1}{2}\begin{pmatrix}1+\eta & \sqrt{1-\eta^2}e^{-i\varphi}\\ \sqrt{1-\eta^2}e^{+i\varphi} & 1-\eta\end{pmatrix}. \tag{4}$$

Conversely, the vector $\mathbf{m}$ is related to the spinor and projector by

$$\mathbf{m}=\langle\mathbf{m}|\boldsymbol{\sigma}|\mathbf{m}\rangle=\mathrm{Tr}(\boldsymbol{\sigma}P_{\mathbf{m}}), \tag{5}$$

where $\boldsymbol{\sigma}$ denotes the three Pauli matrices. The scalar products are related as follows:

$$|\langle\mathbf{m}_1|\mathbf{m}_2\rangle|^2=\frac{1+\mathbf{m}_1\cdot\mathbf{m}_2}{2}. \tag{6}$$

Therefore, orthogonal states are represented by opposite points on the sphere, just as for spins. Among the continuously infinite number of bases, two are more natural: the left $|L\rangle$ and right $|R\rangle$ circular polarization states and the vertical $|V\rangle$ and horizontal $|H\rangle$ linear polarization states (where vertical and horizontal refer to some characteristic direction in the experimental setup). By convention the circular states are mapped to the poles of the Poincaré sphere, and the linear ones are then mapped to the equator (we choose conventions avoiding the use of the imaginary unit i):

$$\mathbf{L}=(0,0,1),\quad |L\rangle=\frac{1}{\sqrt{2}}[|V\rangle+|H\rangle], \tag{7}$$

$$\mathbf{R}=(0,0,-1),\quad |R\rangle=\frac{1}{\sqrt{2}}[|V\rangle-|H\rangle], \tag{8}$$

$$\mathbf{V}=(1,0,0),\quad |V\rangle=\frac{1}{\sqrt{2}}[|L\rangle+|R\rangle], \tag{9}$$

$$\mathbf{H}=(-1,0,0),\quad |H\rangle=\frac{1}{\sqrt{2}}[|L\rangle-|R\rangle]. \tag{10}$$

B. Singlet state for photons

The singlet state (1) for polarization can now be written equivalently in terms of circular or linear polarization states:

$$\begin{aligned}|\psi\rangle&=\frac{1}{\sqrt{2}}[|H\rangle_1|V\rangle_2-|V\rangle_1|H\rangle_2]\\ &=\frac{1}{\sqrt{2}}[|L\rangle_1|R\rangle_2-|R\rangle_1|L\rangle_2].\end{aligned} \tag{11}$$

For photons traveling in optical fibers, two phenomena may affect the polarization correlation: birefringence[13] and polarization-dependent losses (PDL).[14] In practice these effects can be made negligible, but in view of the comparison with the kaon systems, we shall describe them in the following.

C. Birefringence

Birefringence is caused by asymmetries in the fibers. These determine a fast and a slow polarization mode, depending on their relative group velocity. Since these modes are usually linear, we identify them with the vertical and horizontal states, respectively. Birefringence leads to a change of the polarization state. The Poincaré vector $\mathbf{m}(z)$ at position z along the optical fiber rotates around the birefrin-

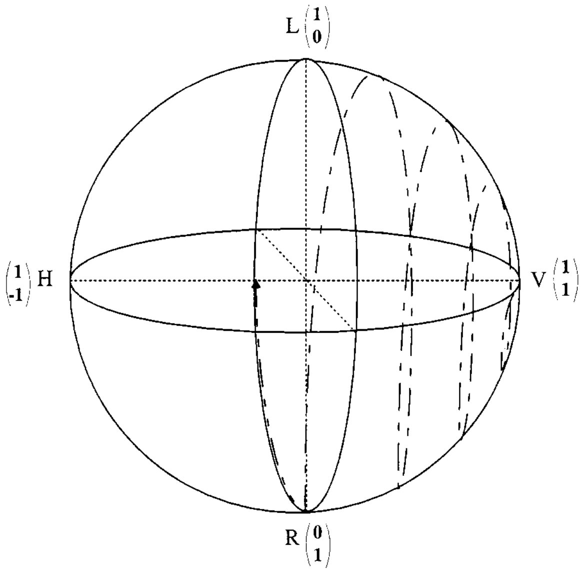

Fig. 2. Poincaré sphere for photon polarization. The double dashed line is the movement of the Bloch vector due to birefringence and the single dashed line is the combined effect of birefringence and PDL [state of non-absorbed photons; Eq. (11)].

gent axis $\boldsymbol{\beta}$. The angular frequency of the rotation depends on the medium and, generally, on the wavelength. For a fixed wavelength one has

$$\frac{\partial}{\partial \dot{z}}\mathbf{m}(z)=\boldsymbol{\beta}\times\mathbf{m}(z), \quad \frac{\partial}{\partial z}|\mathbf{m}(z)\rangle=\frac{-i}{2}\boldsymbol{\beta}\cdot\boldsymbol{\sigma}|\mathbf{m}(z)\rangle. \quad (12)$$

Accordingly,

$$\begin{aligned}&\mathbf{m}(z)=R(\boldsymbol{\beta}(z-z_0))\mathbf{m}(z_0),\\&|\mathbf{m}(z)\rangle=e^{-i(z-z_0)\boldsymbol{\beta}\cdot\boldsymbol{\sigma}/2}|\mathbf{m}(z_0)\rangle,\end{aligned} \quad (13)$$

where $R(\boldsymbol{\alpha})$ is the rotation matrix around the axis $\boldsymbol{\alpha}$ and angle $|\boldsymbol{\alpha}|$, i.e., the rotation corresponding to the unitary operator $U(\alpha)=e^{-i\boldsymbol{\alpha}\cdot\boldsymbol{\sigma}/2}$.

Figure 2 illustrates this rotation in the case of linear birefringence for an axis of rotation $\boldsymbol{\alpha}$ on the equator. [Also shown in Fig. 2 is the trajectory of the Poincaré vector $\mathbf{m}(z)$ when simultaneously affected by polarization-dependent loss, an effect that we study in Sec. III D.]

The effect of birefringence on the singlet state (1) is quite simple. Assuming it acts only on the second photon, it implies that when the first photon is measured to be in a state represented by the Poincaré vector $\mathbf{m}$, then the second photon, instead of being in the orthogonal state $-\mathbf{m}$, is in the ''rotated state'' $-R(\boldsymbol{\beta}(z-z_0))\mathbf{m}$, thus reducing the polarization correlation. However, it is important to notice that this rotated state can be balanced by appropriately counter-rotating the polarization analyzer. Hence birefringence in correlation measurements is merely a matter of aligning polarization analyzers.

D. Polarization-dependent loss (PDL)

Many optical components have polarization-dependent losses (PDL), the most extreme example being a polarizer that completely attenuates one polarization state while not affecting the orthogonal polarization state. Some special fibers also have PDLs.[15] They are made mainly as polarizing elements for fiber sensors. The only case we know where such fibers have been used in a quantum optics experiment was to demonstrate a potentially useful generalized quantum measurement[16] (i.e., not of Von Neumann-type measurement, but an effect sometimes called by the horrible name ''positive operator valued measurement''[17] or POVM).

In the presence of PDL, one state $|+\boldsymbol{\Gamma}\rangle$ undergoes lower losses than its orthogonal state $|-\boldsymbol{\Gamma}\rangle$. Let $T_{\max}$ and $T_{\min}$ denote the maximum and minimum transmission coefficient (for the intensity), respectively. Then the evolution operator T reads, in the $|\pm\boldsymbol{\Gamma}\rangle$ basis:

$$T=\begin{pmatrix}\sqrt{T_{\max}} & 0\\ 0 & \sqrt{T_{\min}}\end{pmatrix}. \quad (14)$$

Note that the transmission coefficient for depolarized light is $T_{\text{depol}}=(T_{\max}+T_{\min})/2$. Arbitrary states $|\mathbf{m}\rangle$ evolve as follows:

$$\begin{aligned}|\mathbf{m}\rangle&=\langle+\boldsymbol{\Gamma}|\mathbf{m}\rangle\cdot|+\boldsymbol{\Gamma}\rangle+\langle-\boldsymbol{\Gamma}|\mathbf{m}\rangle\cdot|-\boldsymbol{\Gamma}\rangle &(15)\\&=P_{+\boldsymbol{\Gamma}}|\mathbf{m}\rangle+P_{-\boldsymbol{\Gamma}}|\mathbf{m}\rangle &(16)\\&\to\sqrt{T_{\max}}P_{+\boldsymbol{\Gamma}}|\mathbf{m}\rangle+\sqrt{T_{\min}}P_{-\boldsymbol{\Gamma}}|\mathbf{m}\rangle &(17)\\&=T|\mathbf{m}\rangle. &(18)\end{aligned}$$

The transmission coefficients are related to the fiber length z:

$$T_{\max}=e^{-\alpha_{\max}z}, \quad T_{\min}=e^{-\alpha_{\min}z}. \quad (19)$$

Hence, the above evolution can be seen as the solution of an evolution equation:

$$\frac{\partial}{\partial z}|\mathbf{m}(z)\rangle=-\left(\tfrac{1}{2}\alpha_{\max}P_{+\boldsymbol{\Gamma}}+\tfrac{1}{2}\alpha_{\min}P_{-\boldsymbol{\Gamma}}\right)|\mathbf{m}(z)\rangle. \quad (20)$$

The evolution of the Poincaré vector $\mathbf{m}(z)$ is more complicated than merely a rotation: While remaining on the sphere, $\mathbf{m}(z)$ tends toward $|+\boldsymbol{\Gamma}\rangle$. Let the $|\pm\boldsymbol{\Gamma}\rangle$ be $(|L\rangle, |R\rangle)$. Figure 2 illustrates such an evolution in the case of a birefringent fiber with PDL (in this example the birefringence axes and the PDL axes $\pm\boldsymbol{\Gamma}$ are the same, as is the case in real fibers).

The effect of PDL on the singlet state (11) is more subtle than birefringence. Assuming it acts only on the second photon, it implies that when the first photon is measured to be in a state represented by the Poincaré vector $\mathbf{m}$, then the second photon, instead of being in the orthogonal state $-\mathbf{m}$, is in a state shifted toward $+\boldsymbol{\Gamma}$. Contrary to the case of birefringence, this shifted state cannot be balanced by appropriately setting the polarization analyzer (actually one possibility would be to insert in front of the analyzer an element with the same PDL but opposite axes such that the two PDL effects result in a polarization-independent loss. But without additional loss the PDL cannot be balanced). Hence PDL in correlation measurements does destroy irreversibly some of the correlation.

IV. THE EPR TEST USING NEUTRAL KAONS

The best example of the massive particle two-state system is the neutral kaon. Neglecting CP violation and for an initial $J^{PC}=1^{--}$ state, where J is the total angular momentum while P and C are parity and charge conjugation discrete symmetries, the wave function can be written in two different bases [similarly to (11)]:

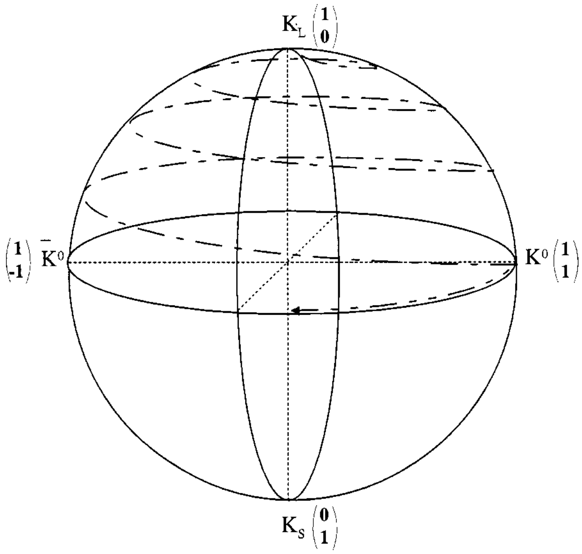

Fig. 3. Poincaré sphere for neutral kaons. The double dashed line is the movement of the Bloch vector due to strangeness mixing alone and the single dashed line is the effect of both strangeness mixing and particle decay (states are re-normalized to the undecayed kaons).

$$|\psi\rangle = \frac{1}{\sqrt{2}}[|K_L\rangle_1|K_S\rangle_2 - |K_S\rangle_1|K_L\rangle_2]$$

$$= \frac{1}{\sqrt{2}}[|K^0\rangle_1|\overline{K^0}\rangle_2 - |\overline{K^0}\rangle_1|K^0\rangle_2], \tag{21}$$

where K_L and K_S are eigenstates of weak interaction while K^0 and $\overline{K^0}$ are eigenstates of strong interaction. This is in exact analogy to the linear and circular polarization states. The transformation rules[18] between the two above bases are formally identical to the photon case (7,...,10), for example:

$$|K^0\rangle = \frac{1}{\sqrt{2}}[|K_S\rangle + |K_L\rangle], \quad |\overline{K^0}\rangle = \frac{1}{\sqrt{2}}[|K_S\rangle - |K_L\rangle]. \tag{22}$$

Strictly speaking, kaons, being spin-0 particles, should not be described by spinors. But, since they do form an isospinor doublet, they can be described like any other two-level quantum system using the Poincaré sphere. Accordingly, the $|K_S\rangle$ and $|K_L\rangle$ states can be represented as points on the north and south poles on the Poincaré sphere, just as the circular polarization states, while $|K^0\rangle$ and $|\overline{K^0}\rangle$ correspond to two opposite points on the equator, just as the linear polarization states $|H\rangle$ and $|V\rangle$, see Fig. 3.

Mixing and decay of the neutral kaon. The kaon time evolution reads:

$$i\hbar \frac{\partial}{\partial t}|K_S\rangle = \left(m_S - i\frac{\gamma_S}{2}\right)|K_S\rangle, \tag{23}$$

$$i\hbar \frac{\partial}{\partial t}|K_L\rangle = \left(m_L - i\frac{\gamma_L}{2}\right)|K_L\rangle, \tag{24}$$

where $m_L(m_S)$ is the mass of the long (short) eigenstate and $\gamma_L(\gamma_S)$ is the width of the long (short) eigenstate. Accordingly, the time evolution operator $U(t)$ is diagonal in (K_S, K_L) and can be decomposed into a (birefringence-like) rotation and a (PDL-like) contraction:

$$U(t) = \begin{pmatrix} \exp\left[-\left(im_S + \frac{\gamma_S}{2}\right)t\right] & 0 \\ 0 & \exp\left[-\left(im_L + \frac{\gamma_L}{2}\right)t\right] \end{pmatrix} \tag{25}$$

$$= \exp\left(-i\frac{m_S + m_L}{2}t\right)\cdot \exp\left(-i\frac{m_S - m_L}{2}t\sigma_3\right) \cdot \begin{pmatrix} \exp(-\gamma_S t/2) & 0 \\ 0 & \exp(-\gamma_L t/2) \end{pmatrix}. \tag{26}$$

The first term in (26) is only a global phase factor. The second term, with dependence on the mass difference between K_L and K_S, produces mixing between K^0 and $\overline{K^0}$ (also called strangeness mixing), an effect formally analogous to birefringence. It is represented by a rotation around the north–south axis of the Poincaré sphere, similar to (12) in Fig. 3. Finally, the third term is due to the fact that neutral kaons are unstable particles, K_S decaying faster than K_L. It is formally identical to Eq. (14), which describes PDL in fibers. It is represented by the precession toward K_L on the Poincaré sphere in Fig. 3.

V. ANALOGIES AND DIFFERENCES BETWEEN THE TWO SYSTEMS

The analogies should now be obvious; compare Figs. 2 and 3. First, both birefringence in optical fibers and the strangeness mixing in neutral kaon produce rotations of the Poincaré vectors representing the quantum states of the photons and kaons, respectively. The only (minor) difference is that conventionally one represents the rotation axis on the equator for photons while the rotation axis for the kaon system passes through the poles (actually this assumes linear birefringence as in real optical fibers; for circular birefringence—also called optical activity—the rotation axis also passes through the poles). Next, PDL in optical fibers, just like kaon decays, produces identical contractions of the state space. As for birefringence, the axes of the contraction differ by convention, but the resulting effects are identical.

Despite these strong analogies, there are also deep differences. First, contrary to polarization analyzers, kaon "analyzers" cannot be simply rotated. Actually, all that can be done is to distinguish (i.e., analyze) between the K^0 and $\overline{K^0}$ by their decay or interaction products. In the optical analogy, this means that we have access to only one position of linear polarization analyzers. Experiments with kaons thus need the rotation produced by strangeness mixing (the apparent birefringence) as a natural way to effectively rotate the kaon analyzer by adjusting its distance to the source! Second, contrary to the PDL of optical fibers, the kaon decay cannot be reduced or compensated. This is the main drawback of EPR tests using kaons. Indeed, the decay, contrary to the strangeness mixing, irreversibly reduces the quantum correlation between the two particles. (Likewise PDL, but not birefringence, reduces the correlation between the photons. But, as already mentioned, this correlation can be recovered at the cost of extra losses.)

Finally, let us also mention the obvious difference: Kaons, contrary to photons, are massive particles. This important difference by itself justifies the interest in using kaons for tests of basic quantum mechanics, such as entanglement. Indeed, it would not be safe to base our confidence in the most peculiar predictions of quantum mechanics, such as quantum nonlocality, on massless particles only.

VI. TESTING THE BELL INEQUALITY

A. The Bell–CHSH inequality

Very briefly, the Bell–CHSH (Clauser–Horne–Shimony–Holt) inequality[19] can be derived as follows. Let α, α' and β, β' denote possible settings (i.e., parameters) of Alice's and Bob's measuring devices[20] (Fig. 1). Assume the results $\mu = \pm 1$ are determined by the local setting and by a global hidden variable $\lambda: \mu(\alpha,\lambda) = \pm 1$ and similarly for the other setting. Note the important physical assumption that the result on Alice's side does not depend on Bob's setting, and vice versa: $\mu(\beta, \lambda)$ is independent of α. This is the locality condition in EPR. The Bell–CHSH inequality is based on the following trivial inequality [note that either $\mu(\beta,\lambda) = \mu(\beta',\lambda)$, or $\mu(\beta,\lambda) = -\mu(\beta',\lambda)$]:

$$\begin{aligned}\mu(\alpha,\lambda)\mu(\beta,\lambda) - \mu(\alpha,\lambda)\mu(\beta',\lambda) + \mu(\alpha',\lambda)\mu(\beta,\lambda) \\ + \mu(\alpha',\lambda)\mu(\beta',\lambda) \\ = \mu(\alpha,\lambda)(\mu(\beta,\lambda) - \mu(\beta',\lambda)) \\ + \mu(\alpha',\lambda)(\mu(\beta,\lambda) + \mu(\beta',\lambda)) \leqslant 2. \end{aligned} \quad (27)$$

Define the expectation value of the product of the outcomes on both sides for settings α and β:

$$E(\alpha,\beta) = \int d\lambda \rho(\lambda) \mu(\alpha,\lambda) \mu(\beta,\lambda), \quad (28)$$

where $\rho(\lambda)$ is an (integrable) probability distribution. The Bell–CHSH inequality follows from (27):

$$S = E(\alpha,\beta) - E(\alpha,\beta') + E(\alpha',\beta) + E(\alpha',\beta') \leqslant 2. \quad (29)$$

B. Bell tests with photons

For photon polarization, the settings are the angles of linear analyzers with respect to some arbitrary origin. For the singlet state (11), by symmetry one has $E(\alpha,\beta) = E(|\alpha - \beta|) = -\cos(\alpha - \beta)$ [Fig. 4(a)]. Actually, it suffices to consider the following settings: $\alpha = 0$, $\alpha' = 2\theta$, $\beta = \theta$, and $\beta' = 3\theta$. The Bell–CHSH inequality (29) depends then only on one parameter:

$$S(\theta) = 3E(\theta) - E(3\theta) \leqslant 2. \quad (30)$$

Figure 5(a) displays the QM prediction of $S(\theta)$. There is a clear violation of the Bell–CHSH inequality over the entire range $0° < \theta < 68.5°$ and a maximal violation by a factor $\sqrt{2}$ for $\theta = 45°$.

In actual experiments, due to the finite efficiency of the detectors, one does not have access to $E(\alpha,\beta)$, and only coincidence rates are measurable, such as $R_{++}(\alpha,\beta)$ for the

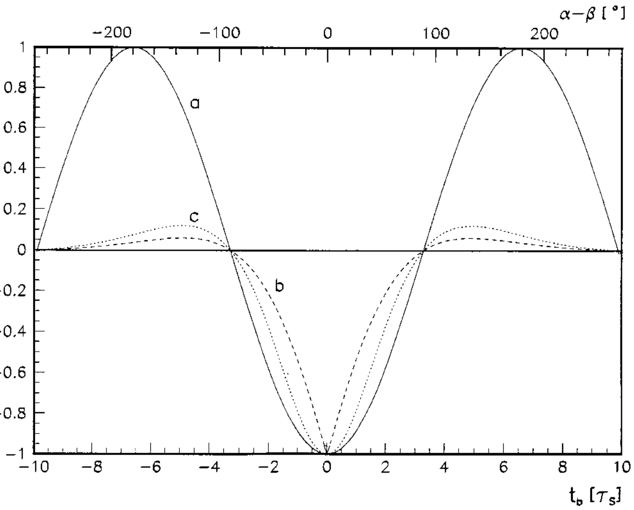

Fig. 4. Correlation function for (a) spin $\frac{1}{2}$, polarized photons and neutral kaon pairs with no decay, (b) kaons with decay [Eq. (33)], (c) kaons with decay and normalized correlation [Eq. (34)].

$(+1, +1)$ coincidence rate. Assuming that the set of detected events constitutes a fair sample,[5] one uses the normalized correlation function

$$E_R(\alpha,\beta) = \frac{R_{++}(\alpha,\beta) + R_{--}(\alpha,\beta) - R_{+-}(\alpha,\beta) - R_{-+}(\alpha,\beta)}{R_{++}(\alpha,\beta) + R_{--}(\alpha,\beta) + R_{+-}(\alpha,\beta) + R_{-+}(\alpha,\beta)}, \quad (31)$$

which is experimentally found to violate the Bell–CHSH inequality.[9–11]

C. Bell tests with kaons

The situation for kaons is similar, except for the already mentioned intrinsic effective birefringence (due to strangeness mixing) and the effective PDL (due to kaon decay). Since the analyzer cannot be rotated, the effective birefringence is used: Delaying the measurement effectively rotates the analyzer. Hence we denote the settings by times t_a and t_b. This would be perfect if there were no decay. But the latter reduces coincidence rates,[12] e.g.:

$$R_{++}(t_a,t_b) = \tfrac{1}{8} e^{-(\gamma_S+\gamma_L)t} (e^{-\gamma_S \Delta t} + e^{-\gamma_L \Delta t} - 2e^{-\gamma \Delta t} \cos[(m_S - m_L)\Delta t]). \quad (32)$$

Accordingly, the correlation function is [Fig. 4(b)]

$$E(t_a,t_b) = -e^{-2\gamma t'} e^{-\gamma \Delta t} \cos[(m_S - m_L)\Delta t], \quad (33)$$

where $\gamma = (\gamma_S + \gamma_L)/2$, $t' = \min(t_a,t_b)$, and $\Delta t = \mathrm{abs}(t_a - t_b)$, $(m_S - m_L \approx 0.477\gamma_S,\ \gamma_S \approx 580\gamma_L)$. This damping makes it impossible to violate the Bell–CHSH inequality[21] [Fig. 5(b)]. However, if one normalizes the correlation function to the undecayed pair of kaons [see (31)], then the correlation function is less damped [Fig. 4(c)]:[22]

$$E_R(t_a,t_b) = \frac{-2e^{-\gamma \Delta t} \cos[(m_S - m_L)\Delta t]}{e^{-\gamma_S \Delta t} + e^{-\gamma_L \Delta t}}. \quad (34)$$

It does violate the Bell inequality, though by less than in the photon case, with a maximum of 2.35 instead of $2\sqrt{2}$ [Fig. 5(c)].[23]

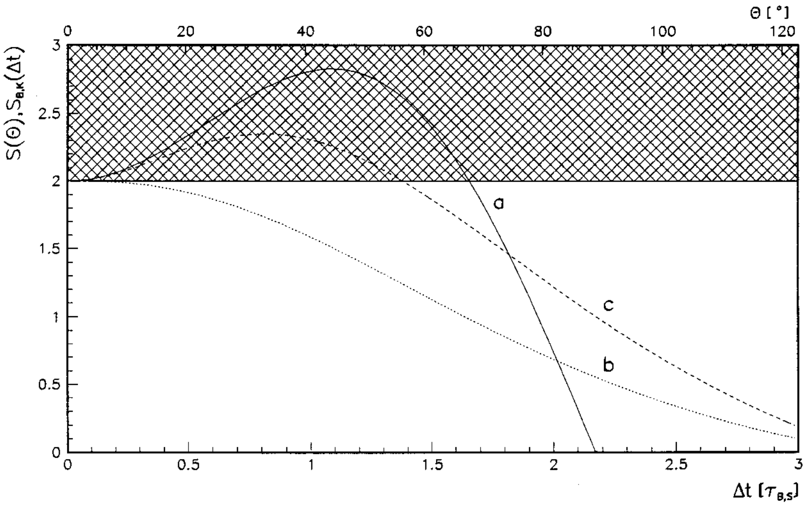

Fig. 5. Bell–CHSH inequality S for (a) spin-$\frac{1}{2}$ and polarized photons, (b) kaons, (c) kaons with re-normalized decay probability. The shaded area violates the Bell inequality. For the B meson, the curve is almost indistinguishable from the spin-$\frac{1}{2}$ case (a), with $\theta = (\Delta M_B / \tau_B)\Delta t = 0.723\Delta t$.

VII. CONCLUSION

The EPR–Bell argument is a beautiful example of simple reasoning leading to deep insight into the nature of physics. It opens the road to experiments involving distant quantum correlated systems and has triggered fruitful philosophical reflections on our world view, including proposals to exploit entanglement for information processing.[24]

We have shown that experiments with kaons have striking analogies with the most studied case of photons.

• Photons have the advantage of being easy to produce and to propagate over long distances, but the drawback is that they are difficult to detect efficiently. Hence, to date all Bell tests using photons are based on the normalized correlation function $E_R(\alpha,\beta)$ defined in (31). Kaons, by contrast, are massive particles, in principle easier to detect by their decay products. However, due to their intrinsic decay rate, one still has to base Bell tests on the normalized correlation function $E_R(\alpha,\beta)$.

• The kaon decay is formally analogous to polarization dependent losses (PDLs) for photons, but it carries a very different meaning: While PDL is due to external parameters over which the experimenter has control (by changing the fiber, for example), the decay is an intrinsic property of the kaon over which the experimenter has no control. This puts a severe limit on Bell tests; they cannot rule out any local theory where the kaon decay is pre-determined. In other words, such a Bell test is not loophole free.

• As for the relevant intrinsic property of kaons, strangeness mixing is analogous to optical birefringence. Contrary to the decay, strangeness mixing is an advantage for Bell tests, as it allows one to effectively rotate the analyzer simply by delaying the measurement.

Finally, let us mention another interesting massive particle system: the B meson. A pair of $B^0\overline{B}^0$ created at $\Upsilon(4S)$ resonance has exactly the same formalism as the $K^0\overline{K}^0$ but with a major difference: $\gamma_S \approx \gamma_L$. As a consequence, the exponential terms in (34) cancel out and the correlation function is again sinusoidal with the maximum again at $2\sqrt{2}$, just as for the photons [Fig. 5(a)]. In fact, such a maximal violation of the Bell–CHSH inequality could be tested experimentally at the asymmetric B factories: BELLE in KEK, Japan[25] and BaBar in SLAC, USA.[26]

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SPONGE THEORISTS

Consider what you must believe or at least be perplexed about if you uncritically accept that air is a sponge with a carrying capacity that increases with temperature:

1. At a relative humidity of 100% all pores in the air sponge are filled with water vapor. Therefore, gasoline, benzene, alcohol, and thousands of other volatile compounds cannot evaporate into air with 100% relative humidity because there is no room for the molecules of these compounds.
2. Air of a given temperature above a solution holds less water vapor than air of the same temperature above pure water. Thus air somehow knows when it is above a solution and shrinks its pore sizes accordingly.
3. The air around a small droplet holds more water vapor than that around a large droplet (in Section 5.10, we'll make clearer what is meant by ''small'' and ''large''). Thus air somehow can sense the size of droplets and shrink or expand its pores accordingly.
4. Although air is a sponge that can hold only so much water vapor, by some mysterious process it sometimes can increase the size and number of its pores. This is the phenomenon of *supersaturation*, an essential ingredient in the formation of clouds (see Section 5.11). Adherence to the sponge theory of air makes it difficult to understand (or even accept) supersaturation.

Confused? If so, don't blame us; blame the scores of sponge theorists who have thoughtlessly passed on nonsense dressed up as knowledge.

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